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Range Glint Effects for Homing Interceptor Miss Distance

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FOR THE COMMANDER

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RANGE GLINT EFFECTS FOR HOMING INTERCEPTOR MISS DISTANCE

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Group 32

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ABSTRACT

An expression is derived for the single hit range error due to the presence of a secondary scatterer in a range measurement made by a single frequency radar using gaussian pulses and a centroid estimator. The effect of noise is neglected since glint effects are only important at close range. These results are used to calculate the effects of glint on a homing radar which tracks the target for several hits. Contours of equal miss distance are presented for the two cases of decorrelated (random) phase between hits, and uniform phase advance.

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1. INTRODUCTION

A radar return from a complex object which has more than one scattering center will consist of a train of pulses for each transmitted pulse. If the bandwidth of the radar is high, and the length of a pulse is small compared to the spacing between pulses in the train, then the individual pulses will be resolved, and the radar will be able to see the structure of the object, and track on whichever feature is desired.

When the radar bandwidth is small, the pulse width is large compared to the spacing between pulses, and individual pulses cannot be resolved. A measurement of range taken from this composite return will yield a result which depends on:

1) the algorithm used in the signal processor
2) the shape of the radar pulse
3) the bandwidth of the pulse
4) the operating wavelength of the radar
5) the number and nature of the scattering centers on the target
6) the shape of the target, which determines the relative position of the scattering centers
7) the relative scattering power (radar cross-section) of the scatterers
8) the target orientation.

Factors (1) through (4) are properties of the radar,

and will therefore be known. The remainder are properties of the target, and will not in general be known -- the last two factors may also vary with time.

This report will present an analysis of the range errors introduced by multiple reflections, and the effect these errors have on the accuracy of tracking an object on to which the radar is homing. The effect of noise on the received signal will be ignored, since the errors due to multiple reflections are small enough to be important only at close range, when the signal-to-noise ratio will be large.

Much of this analysis may be applicable to the case of angle estimation using monopulse techniques. However, this report considers only range estimation.

2. SINGLE HIT RANGE ERROR

To simplify this analysis, the target will hereafter be assumed to consist of two point scatters, separated by a distance L along the line of sight to the radar, which uses single frequency gaussian shaped pulses, and a centroid estimator in the signal processor. When such a processor is given an input which consists of two gaussian pulses, the range estimate it returns is a function of the separation, width, amplitude ratio and relative phase of the pulses. This function will now be evaluated.

2.1 Centroid of Two Gaussian Pulses

We have two returned pulses, whose amplitudes and phases vary as

$$A_1 \, e^{-\left(\frac{t^2}{\tau^2}\right)} e^{j\omega t} \tag{1}$$

$$A_2 \, e^{-\left(\frac{(t-D)^2}{\tau^2}\right)} e^{j(\omega t+\phi)} \tag{2}$$

Where: time is measured from the arrival of the peak of pulse 1

D is the time delay between pulses

τ is a measure of the pulse width

ϕ is the relative phase of the returns

ω is the angular frequency of the radar carrier

The modulus square of the sum of these two pulses is the received power, and is

$$I(t) = A_1^2 e^{-\left(\frac{2t^2}{\tau^2}\right)} + A_2^2 e^{-\left(\frac{2(t-D)^2}{\tau^2}\right)} + 2A_1 A_2 \cos\phi \, e^{-\left(\frac{t^2}{\tau^2} + \frac{(t-D)^2}{\tau^2}\right)}$$

The centroid of I(t) over time is equal to

$$\langle t \rangle = \frac{\int_{-\infty}^{\infty} t.I(t)\, dt}{\int_{-\infty}^{\infty} I(t)\, dt}$$

The integrals are a little cumbersome, but putting $A_2/A_1=A$ it can be shown that

$$\langle t \rangle = \frac{\frac{\pi}{2} e^{-\left(\frac{D^2}{2\tau^2}\right)} \left[A^2 D\tau\, e^{\frac{D^2}{2\tau^2}} + \cos\phi\, A\, D\tau\right]}{\frac{\pi}{2} e^{-\left(\frac{D^2}{2\tau^2}\right)} \left[(A^2+1)\, \tau\, e^{\left(\frac{D^2}{2\tau^2}\right)} + 2\, A\tau \cos\phi\right]}$$

$$\Rightarrow \langle t \rangle = D\, A \left[\frac{Ae^V + \cos\phi}{(A^2+1)\, e^V + 2A\cos\phi}\right] \quad \text{where } V = \frac{D^2}{2\tau^2}$$

Now putting $w^2 = 2\tau^2$ then the amplitude of signal 1 will be $A_1 e^{-\left(\frac{2t^2}{w^2}\right)}$, and the power will vary as $e^{-\left(\frac{4t^2}{w^2}\right)}$. For $t = \pm w/2$, the power drop from the peak will be close to 3 db (actually ≈ 4dB) and so w is the pulse width. To convert the estimate of time $\langle t \rangle$ to an estimate of range $\langle r \rangle$ we say:

$$\langle r \rangle = \frac{\langle t \rangle}{2} c \quad (c = \text{velocity of light})$$

$$D = 2 \times \text{line of sight separation between scatters } (L)/c$$
$$= 2L/c$$

$$w = \text{time width of pulse} = \text{spatial width of pulse } (P)/c$$
$$= P/c$$

$$\langle r \rangle = \frac{c}{2} \cdot \frac{2L}{c} \cdot A \frac{Ae^V + \cos\phi}{(A^2+1)\, e^V + 2A\cos\phi} \text{ with } V = \frac{D^2}{2\tau^2} = \frac{D^2}{w^2} = \left(\frac{2L}{P}\right)^2$$

so

$$\Rightarrow \langle r \rangle = L\, A \frac{A\, e^V + \cos\phi}{(A^2 + 1)\, e^V + 2A\cos\phi} \quad \text{with } V = \left(\frac{2L}{P}\right)^2$$

This is the estimate of range relative to the true position of scatterer #1, so if we are trying to track on #1, then $\langle r \rangle$ is the size of the error.

It is convenient to introduce a variable E, the normalized range error given by $\langle r \rangle/L$, and hence

$$E = A \frac{Ae^V + \cos\phi}{(A^2 + 1) e^V + 2A \cos\phi}$$

Using this parameter, if E = 0 then the estimate corresponds to the position of the closer scatterer; if E = 1, the more distant scatterer.

In Fig. 1 is shown E plotted as a function of A (amplitude ratio, $= A_2/A_1$) for several different bandwidths and the two extremes of phase ($\phi=0,\pi$). Figure 2 shows E as a function of phase angle for several different bandwidths at a typical amplitude ratio of A = 0.7.

From the figures several features may be noted:

(a) It is possible for the position estimate to fall outside the physical bounds of the object (E = 0 to E = 1).

(b) The largest errors occur for phase angles close to π, narrowband pulses ($P \gg L$) and amplitude ratios close but not equal to unity.

(c) If the two scattered amplitudes are the same (A = 1), the position estimate is midway between the two scatterers (E = 0.5), independent of phase angle or pulse bandwidth.

(d) If one center scatters much more strongly than the other, the apparent position will be close to that scatterer.

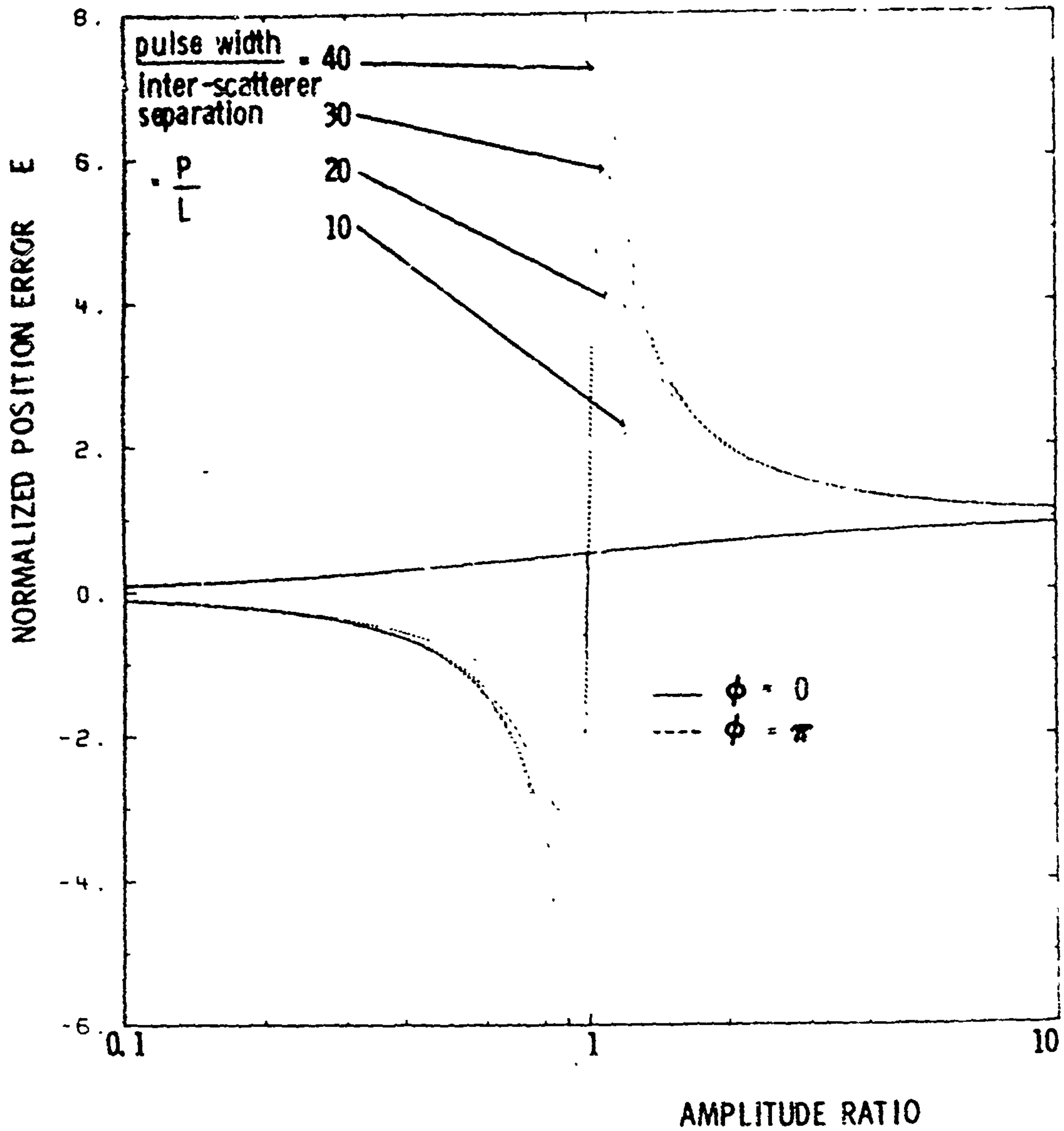

Fig. 1. Normalized position error as a function of return pulse amplitude ration for four values of (pulse width/inter-scatterer distance) and two values of relative phase.

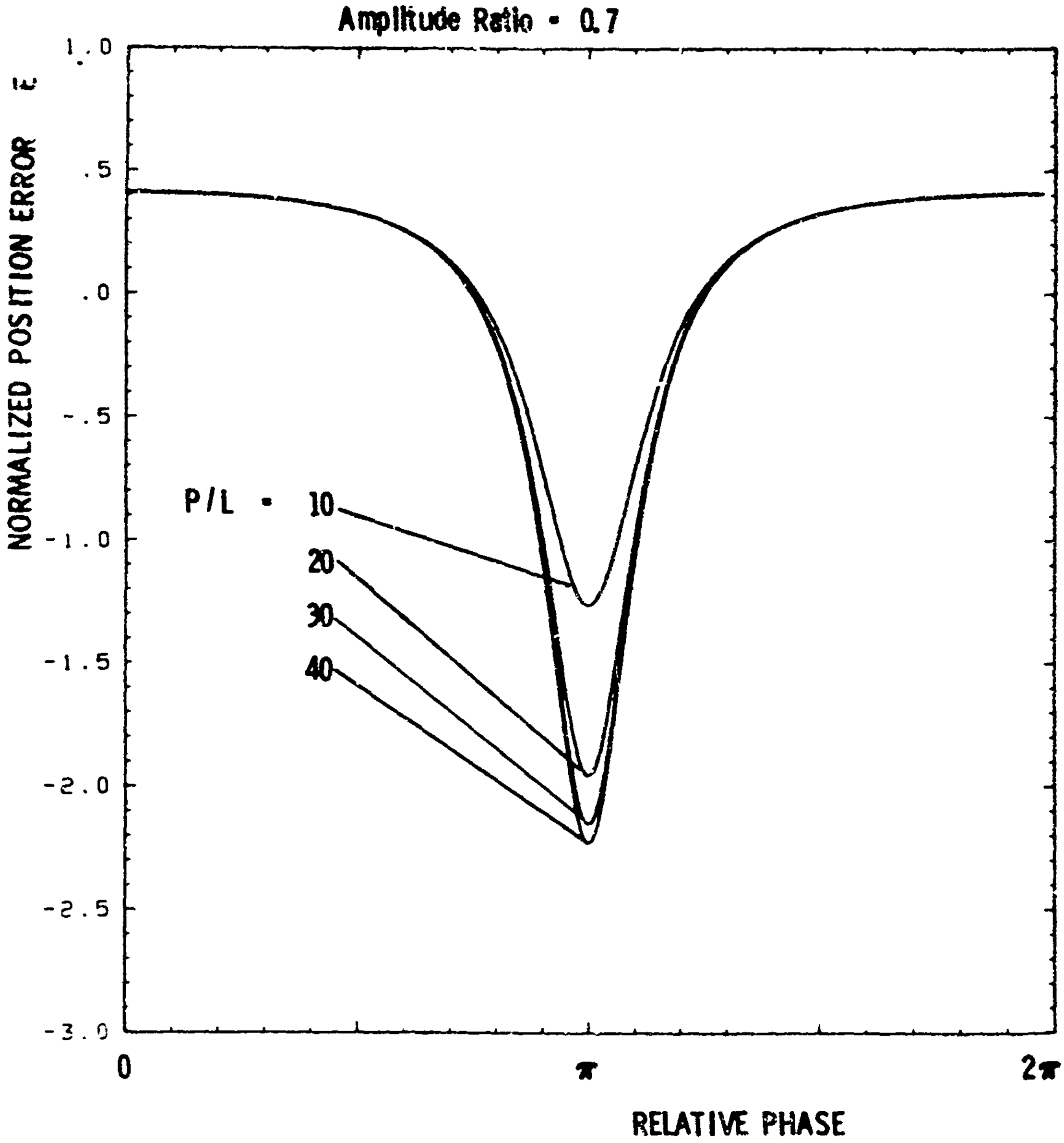

Fig. 2. Normalized position error as a function of relative phase between returns for four values of (pulse width/inter-scatterer distance), and an amplitude ratio of 0.7.

3. TRACKING

When a radar is used to track a moving object, it will normally make several measurements on the object, and perform a fit on these measurements to obtain a trajectory. A typical example of a case where a tracking radar is homing in on a target is given in Fig. 3a, which shows the true trajectory, the range estimates made by the radar, and the apparent trajectory produced by a linear fit to the range estimates. Due to the errors on the range measurements, the apparent trajectory is incorrect, and predicts zero range (impact) at the point T_O when in fact, the object is still a distance M away. This distance M will depend on the number of range measurements made, the time over which they are made (the track time T_t), the time between the last measurement and impact (the predict time T_p), and the distribution of the range errors.

Figure 3b shows a similar engagement but with the true range subtracted for clarity, and so the vertical axis represents (estimated range-true range) = range error. The miss distance M on Fig. 3b comes from two sources: a bias due to the non-zero mean position error, and a spread dependent on the range of values of the position error, which causes the apparent velocity to be in error. The latter shows up as a fit

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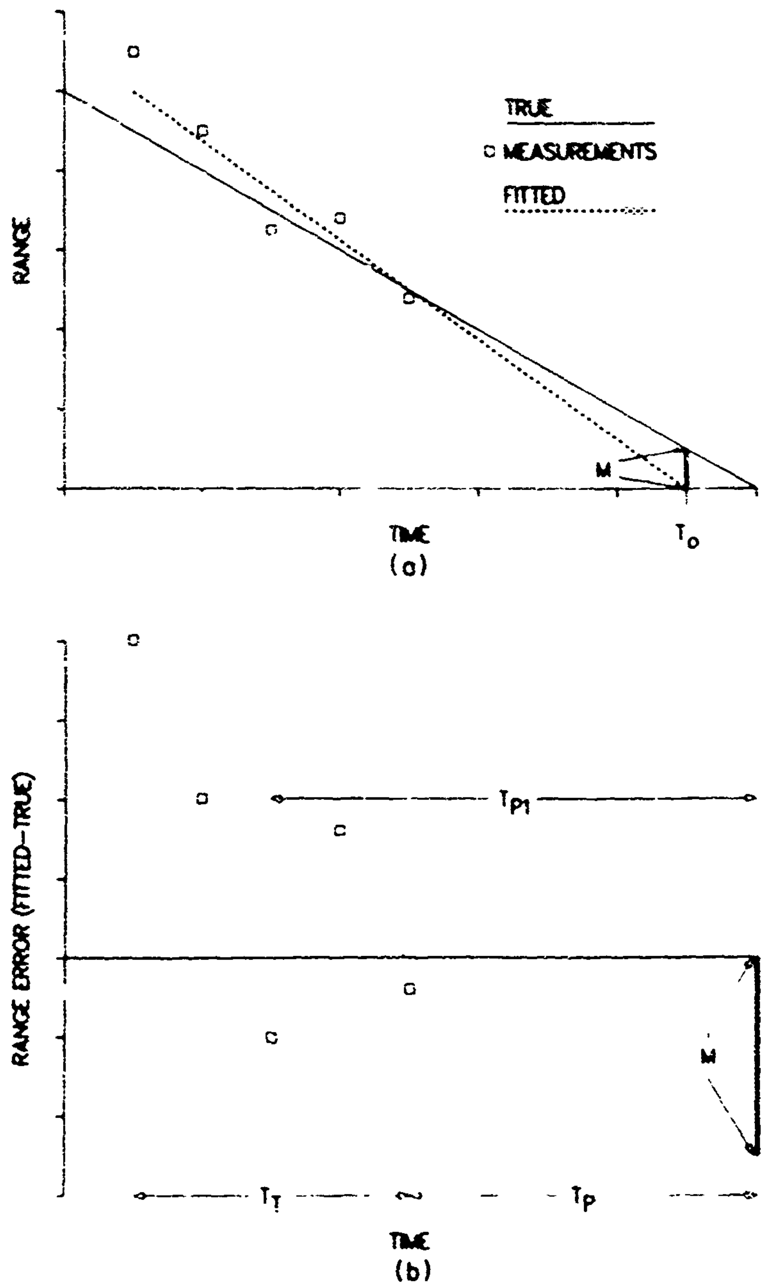

Fig. 3 A tracking and homing engagement showing the true range of the target, measurements of range which are subject to error, and the estimated trajectory produced by fitting a straight line through the measurements: (a) Ranges shown measured from the tracker, (b) Ranges shown relative to the true range.

line of non-zero gradient in Fig. 3b. It will be apparent that the miss distance due to bias does not depend on T_p or T_T, but the miss distance spread due to apparent velocity errors will be proportional to $\frac{T_{p1}}{T_T}$. Using the fact that $T_{p1} = T_p + \frac{T_T}{2}$, the miss distance spread will be proportional to $\frac{T_p + T_{T/2}}{T_T}$.

When the radar wavelength is much smaller than the dimension of the object (as will usually be the case), the single-hit parameter likely to change the most between hits is the phase. For ease of analysis, it will be assumed hereafter that the phase is the only parameter to change over the track time.

Two cases will be considered; one in which the phase changes randomly from pulse to pulse and a second in which the phase changes uniformly. If the object is spinning or tumbling in a quasi-random fashion (such as may be induced by atmospheric turbulence) or the radar frequency is changed between hits (frequency hopping). then there will be no correlation between phases at successive observations, and the phase may be regarded as a uniform random variate in the range $0 \rightarrow 2\pi$.

With this assumption the mean value or bias of the

normalized miss-distance (the miss distance/scatterer separation) = M/L is equal to the normalized position error, averaged over all phases.

A plot of the mean value of normalized miss distance is shown in Fig. 4 as a function of amplitude ratio for several bandwidths. It will be noted that the mean value is close to zero for $A \ll 1$ (average miss distance = 0), and unity (average miss distance = separation of scattering centers) for $A \gg 1$. This is intuitively obvious: if the secondary scatterer has a much stronger return than the primary scatterer, then the apparent position of the object will always be close to the secondary scatterer; if also the return is assumed to be for the primary scatterer, then the range will be in error by the inter-scatterer separation. Provided one knows what part of the object one is tracking on, the miss distance due to bias will be small.

A number of Monte-Carlo type simulations were run to find the variation in the spread (one standard deviation) in the normalized miss distance with amplitude ratio and pulse width. Ten position estimates were made in each case, and the phase was selected at random in $0 \rightarrow 2\pi$.

Figure 5 shows a contour plot of the standard deviation of the normalized miss distance M/L as a function of amplitude ratio and pulse width for $T_P = \tau_{T/2}$. The actual miss

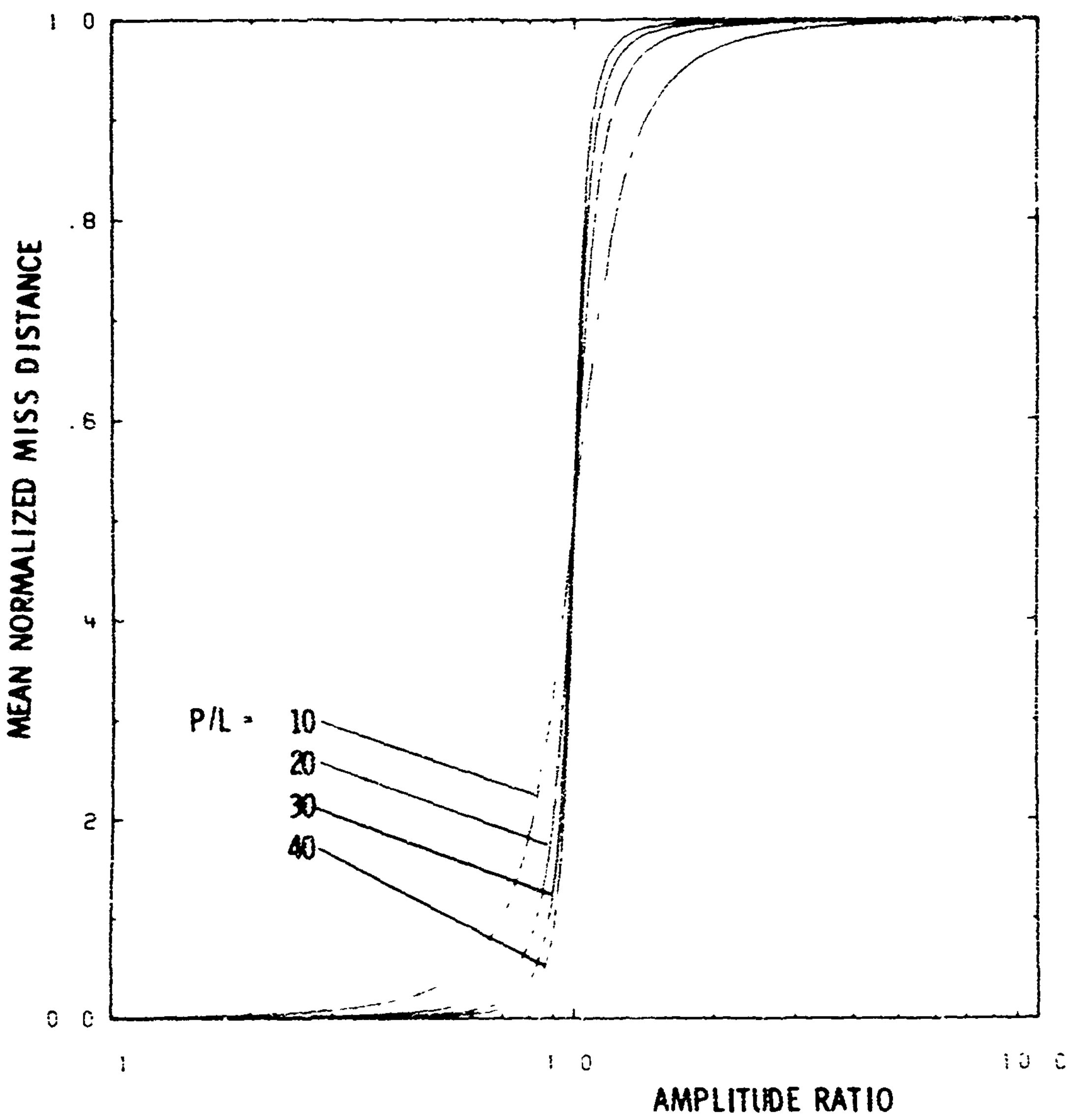

Fig. 4. Normalized miss distance averaged over phase, as a function of amplitude ratio and (pulse width/inter-scatterer distance)

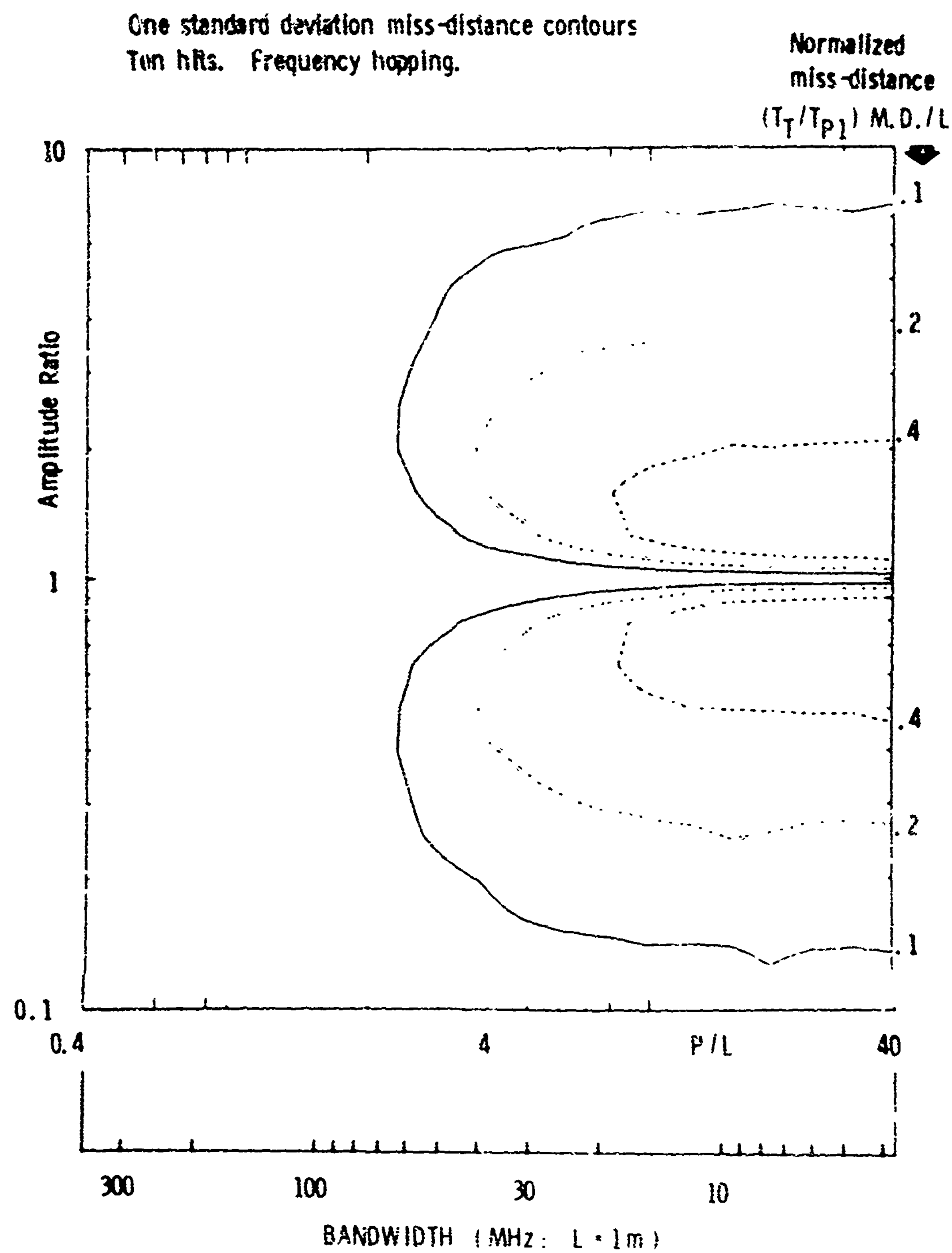

Fig. 5. Contours of equal normalized miss distance as a function of amplitude ratio and bandwidth, for ten uncorrelated measurements.

distance spread increases with increasing predict times T_p and values of the contours should be multiplied by $\frac{T_p + T_{T/2}}{T_T}$ to to find M/L, and by L (inter-scatterer distance) to find the actual miss distance.

As with the single hit error, the worst miss distances occur for amplitude ratios near unity and narrowband radars (P/L>>1).

If a value is assigned to the inter-scatterer distance, then the horizontal axis of Fig. 5 can be re-labelled with radar bandwidth in Megahertz, using the relationship (pulse width in meters) x (bandwidth in MHz) = 150. This has been done using an assumed distance of 1 meter for L, and the result is shown beneath the original axis. For target dimensions different from 1 meter, the numbers on the Megahertz scale must be multiplied by a factor of 1/L.

Using a constant radar frequency on an object in uniform motion (e.g. in space) may cause the phases of successive returns to be correlated. A typical example of this is shown in Fig. 6, in which the spinning motion of the conical target causes a smooth periodic variation in the inter-scatterer distance. In this case it is not possible to predict exactly what errors to expect, but under certain circumstances

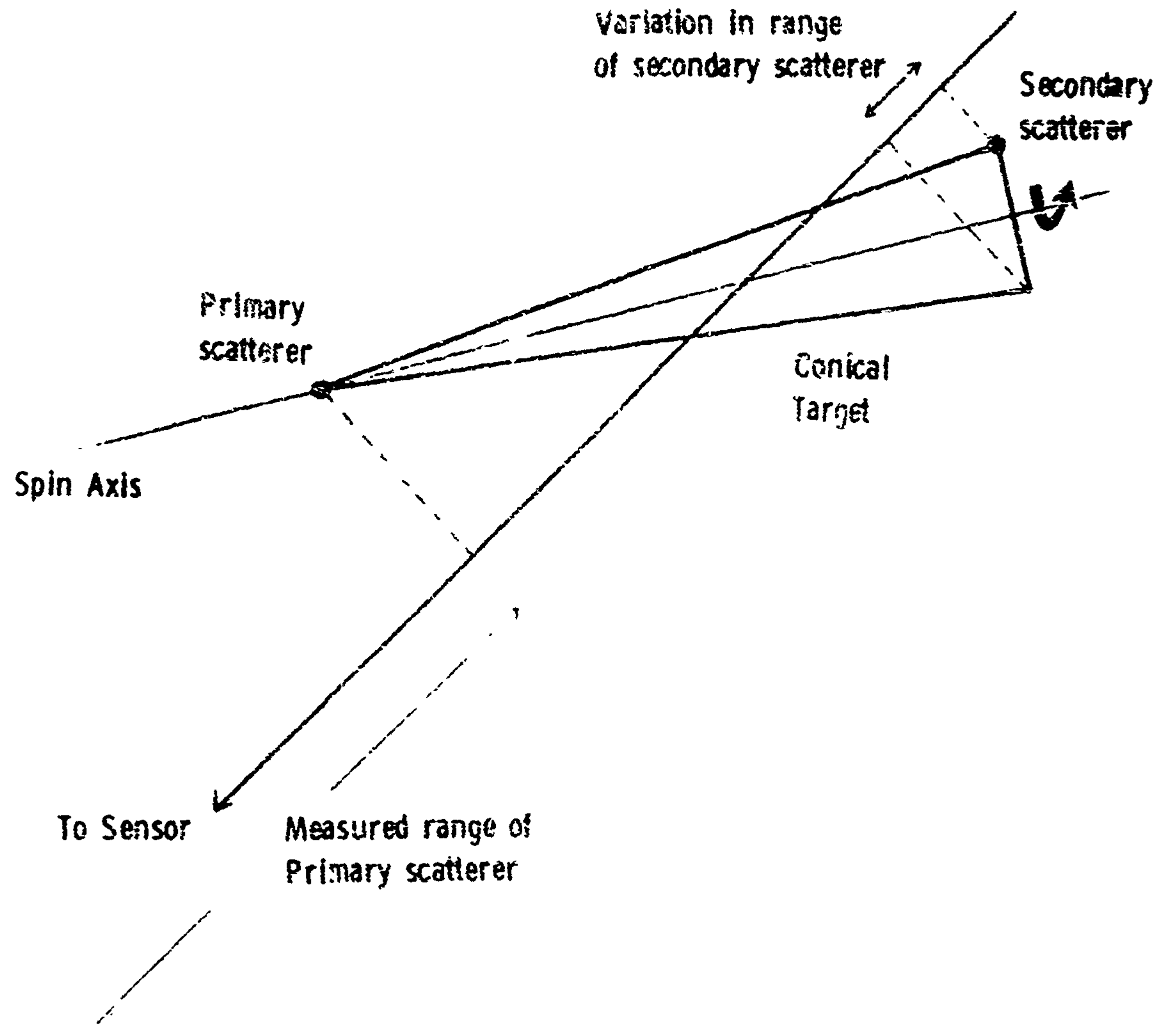

Fig. 6. Example of a target which will yield a smooth regular variation in phase between return pulses from successive measurements.

the miss distance may be very large. If the phase change over the track time is close to $(2n + 1)\pi$ for integer n, then a large velocity error can result, since the position error can be negative at one end of the track, and positive at the other. Figure 7 shows the wide variation in miss distance which can occur with two very similar phase advance rates.

The miss distance contours for the worst case of Fig. 7 were plotted, and are shown in Fig. 8 for the same range of amplitudes and bandwidths used in Fig. 5. It will be noticed that the worst case error is much larger than the RMS error in the frequency hopping case.

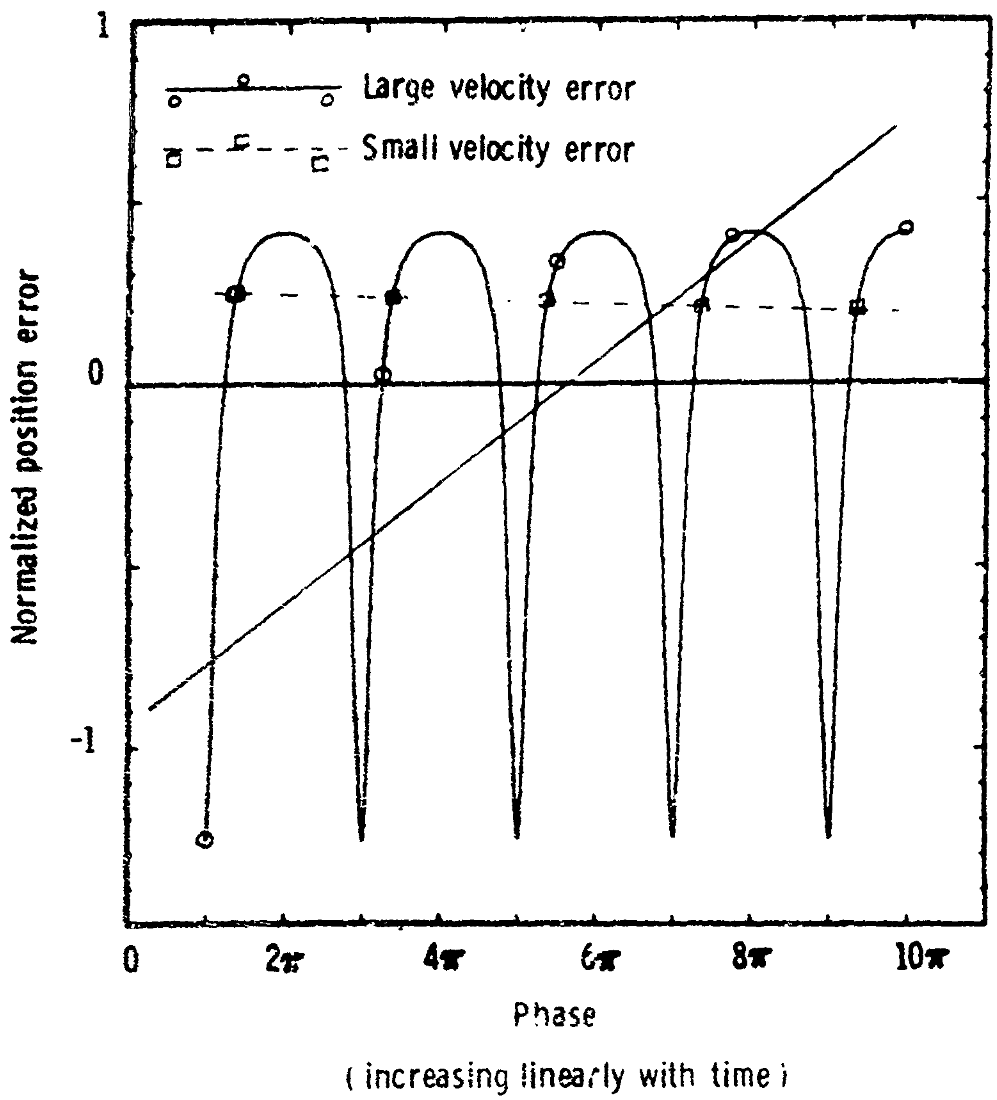

Fig. 7. Example showing how small differences in phase advance rates can lead to large differences in estimated velocity error for linear phase advance (correlated phases).

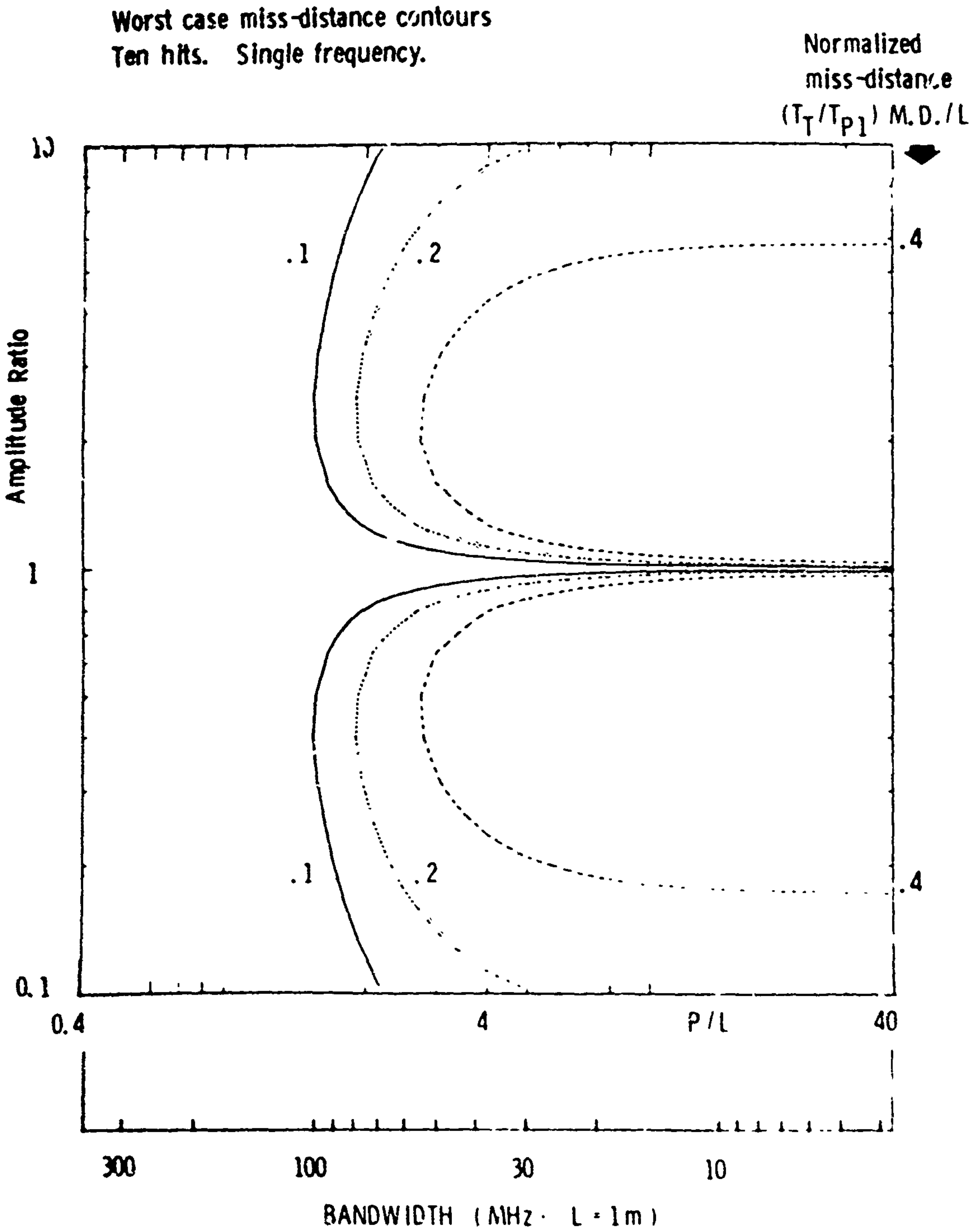

Fig. 8. Contours of equal normalized miss distance as a functiuon of amplitude ratio and bandwidth, for a smooth phase variation of π over the track time (correlated phases).

4. SUMMARY

An expression has been found for the error due to a secondary scatterer in a range measurement made by a single frequency radar using gaussian pulses and a centroid estimator. Under certain circumstances the apparent position may fall outside the physical bounds of the target. Narrowband radar pulses lead to larger errors than wideband (short duration) pulses. Curves showing the behavior of the error as a function of bandwidth, relative scatterer cross-section and inter-pulse phase have been presented.

Monte-Carlo simulations were used to analyze the effect of these errors on a homing radar which tracks for the target and then predicts the miss distance. These analyses neglect any thermal noise in the measurements and thus consider only the effect of glint.

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An expression is derived for the single hit range error due to the presence of a secondary scatterer in a range measurement made by a single frequency radar using gaussian pulses and a centroid estimator. The effect of noise is neglected since glint effects are only important at close range. These results are used to calculate the effects of glint on a homing radar which tracks the target for several hits. Contours of equal miss distance are presented for the two cases of decorrelated (random) phase between hits, and uniform phase advance.

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